On the appearance of swirl in a confined sink flow

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Radial and swirl velocity components in an axisymmetric sink flow inside a cylindrical container of variable aspect ratio H/R are measured using laser-Doppler anemometry. It is shown that a vortex, superimposed to the meridional sink flow, is formed when the Reynolds number based on the sink flow rate is above a critical value. Both clockwise and counterclockwise directions of rotation are found. © *2000 American Institute of Physics.* [S1070-6631(00)01111-9]

Several experimental studies have been carried out in the past to analyze the origin of the bathtub vortex, i.e., the swirling motion observed when a liquid is drained from a hole at the bottom of a tank.[1–4] Kawakubo *et al.*[5] measured the azimuthal velocity component at a given point of the flow by laser-Doppler anemometry (LDA) and showed that the vortex formed when the sink flow rate exceeded a certain threshold value. They concluded that the formation of the vortex was due to a kind of phase transition.

To avoid the effect of the liquid–air free surface, and of the air-entrainment into the vortex, whose influence on the formation of the vortex is difficult to quantify, we have performed a series of experiments in a cylindrical container of variable aspect ratio H/R, where $R=97$ mm is the inner radius of the cylinder and H its height, adjustable to a maximum of about 200 mm [see Fig. 1(a) for a sketch of the experimental apparatus]. The cylindrical tank is closed except for an orifice (the sink) of diameter $d=4.5$ mm centered at its bottom endwall, from where the liquid is drained by a centrifugal pump, and a circular ring between the upper endwall and the cylinder (the upper endwall radius is about 93.5 mm), through which the liquid is reinjected to the tank after passing through a settling chamber. The flow rate Q is measured using a magnetic inductive flow meter, with an uncertainty of about 0.1 l/h, while the temperature is measured with a digital thermocouple in the settling chamber, with an uncertainty of 0.1 °C (the temperature is used to calculate the kinematic viscosity ν of the liquid during the experiments). Since $d \ll R$, the two main parameters governing the flow are the aspect ratio H/R and the Reynolds number

$$\mathrm{Re}=\frac{4Q}{\pi d \nu}. \tag{1}$$

The working fluid is a (72%)-glycerine–(28%)-water solution with a measured index of refraction of 1.437, so chosen to approximately match the index of refraction of the transparent material (Plexiglas) of the container cylindrical wall. This minimizes the refraction of the laser beams of the LDA system (1 component, 10 mW He–Ne laser with wavelength of 632.8 nm manufactured by Dantec) used to measure the fluid velocity inside the cylinder. Salt (NaCl) was also added to the solution to allow for the visualization with hydrogen bubbles using a relatively low electrical potential applied to two very thin wires (0.1 mm of diameter) fixed diagonally at the bottom endwall at both sides of the orifice. This visualization technique was used to detect the swirl in preliminary experiments, but was never used simultaneously with the LDA. The LDA probe is set perpendicular to a glass plane attached as a window to the cylinder, with the space between the plane and the cylinder filled with the same working fluid. The two laser beams are on a horizontal plane at a distance z to the bottom endwall ($z=6$ mm in all the results reported below), intersecting in a point with coordinates (x, y) [or (r,θ)] on that plane, in such a way that the y velocity component is measured [see Fig. 1(b)].

In a typical experiment, for given H/R, Re and z, the coordinate y is fixed ($y=y_0$, say), and the coordinate x is varied between $-x_0$ and $+x_0$ using a traversing system controlled by the LDA software. Assuming that the flow is axisymmetric, the measured v_y is related to the radial (u) and azimuthal (v) velocity components by [see Fig. 1(b)]:

$$v_y(x,y_0)=u(r)\cos\theta - v(r)\sin\theta = u(\sqrt{x^2+y_0^2})\,\frac{y_0}{\sqrt{x^2+y_0^2}} - v(\sqrt{x^2+y_0^2})\,\frac{x}{\sqrt{x^2+y_0^2}}. \tag{2}$$

Since the first term in the right-hand side of (2) is an even function of x, while the second term is an odd function of x, one may obtain $u(r)$ and $v(r)$ separately from $v_y(x,y_0)$ by adding and subtracting, respectively, $v_y^+=v_y(x\geqslant 0,y_0)$ and $v_y^-=v_y(x\leqslant 0,y_0)$ (of course, for $r\geqslant y_0$). Their absolute values are

$$|u(r)|=\frac{r}{2y_0}|v_y^+ + v_y^-|,\quad |v(r)|=\frac{r}{2x^+}|v_y^+ - v_y^-|,$$
$$r=\sqrt{x^{+2}+y_0^2}. \tag{3}$$

The azimuthal velocity could also be obtained directly from v_y by measuring it at $y_0=0$. However, the velocity near the axis $r=0$ is so small that the accuracy of the LDA measurement becomes poor. In addition, with $y_0\neq 0$ one obtains both

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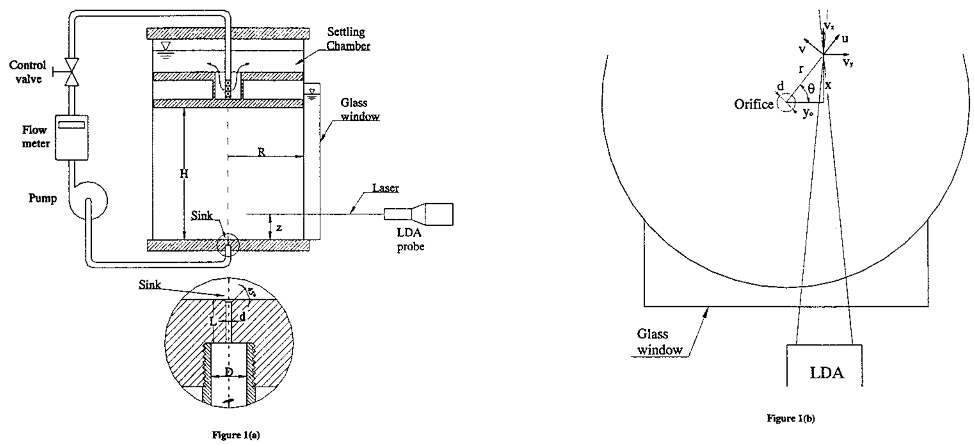

FIG. 1. Experimental setup and coordinate system.

the radial and the azimuthal velocities together (for $r \geqslant y_0$). We used values of y_0 between 1 and 2 mm. If the measured profile $v_y(x,y_0)$ is symmetric in relation to $x=0$, the sink flow is purely radial (it has no rotation). But if the measured profile becomes nonsymmetric, it is an indication that a vortex has been formed superimposed to the sink flow. Previously to the LDA measurements, we performed a series of experiments using the hydrogen bubbles visualisation technique. For several values of the aspect ratio, we found that the swirl appeared in the sink flow for Re above a critical value somewhere between 1300 and 1400.

Figure 2 shows the measured v_y at $y_0 = 1.98$ mm for decreasing [Fig. 2(a)] and increasing [Fig. 2(b)] values of the Reynolds number and $H/R = 0.515$. The experiment was started at the highest value of the Reynolds number [Re $=1848$ in Fig. 2(a)], at which the v_y profile is clearly nonsymmetric in relation to $x=0$, so that a vortex is present in the sink flow at this Re. The rotation in this particular vortex is counterclockwise when seen from above. Then, the Reynolds number was decreased by closing the control valve and thus diminishing the flow rate Q. The corresponding v_y profiles in Fig. 2(a) show that the intensity of the vortex decreased, until it vanished before reaching the last value Re $=1239$ [the profile for a much lower Reynolds number, Re $=749$, has been added to Fig. 2(a) for comparison sake]. After this, the Reynolds number was increased by opening the control valve [Fig. 2(b)], and the swirl appeared again. The radial and azimuthal velocity profiles as functions of r are extracted from the v_y profiles by using (3). Figure 3 shows the maximum values of the azimuthal velocity v as functions of Re for both decreasing (circles) and increasing (squares) Reynolds numbers. A linear interpolation of these data (continuous and dashed lines in Fig. 3, respectively) shows that the critical Reynolds numbers for the disappearance and the appearance of the swirl are practically the same within the experimental errors (we obtain Re$=1381$ and Re $=1367$, respectively).

Although the results given in Figs. 2 and 3 are for a

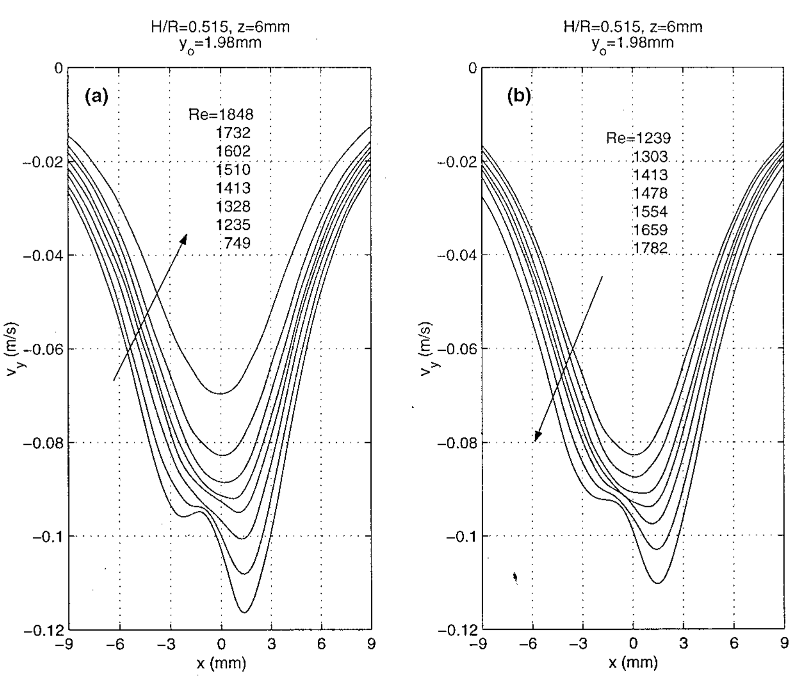

FIG. 2. Measured velocity profiles at $y = 1.98$ mm, $z = 6$ mm, for $H/R = 0.515$ and several Re (a) Decreasing Re. (b) Increasing Re.

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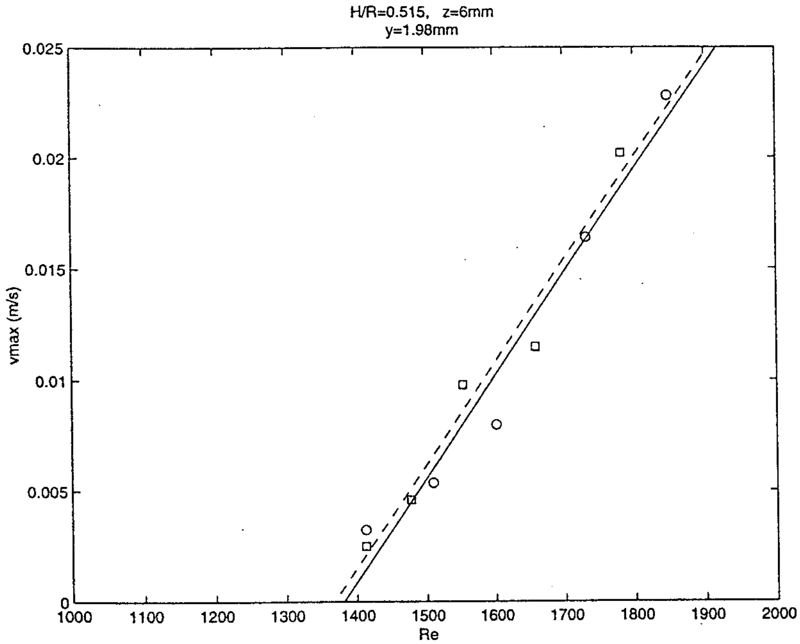

FIG. 3. Maximum swirl velocity for decreasing (circles and continuous line) and increasing (squares and dashed line) Reynolds numbers. $H/R=0.515$, $z=6$ mm.

counterclockwise rotation, we have also measured clockwise rotations for the same value of the aspect ratio $H/R=0.515$. Once a series of measurements was stopped and the flow completely settled down, the vortex developed above the critical Reynolds number in the next experiment could have the same, or the opposite, direction.

The measurements were repeated for several values of the aspect ratio. Figure 4 shows the maximum swirl velocity as a function of Re for $H/R=0.40$ and 0.286 (the experiments shown in these two cases are for decreasing Re), together with the above commented results for $H/R=0.515$. The rotation in the new cases is clockwise (negative values in Fig. 4). The critical Reynolds numbers for the appearance of the swirl ranged between 1300 and 1400, in agreement with the values obtained previously using visualization techniques. They do not seem to depend much on the aspect ratio H/R within the experimental errors of the measurements. As H/R decreased, the intensity of the formed vortex also decreased, thus becoming more and more difficult to accurately measure the azimuthal velocity with the present LDA technique.

The critical Reynolds numbers obtained here are slightly

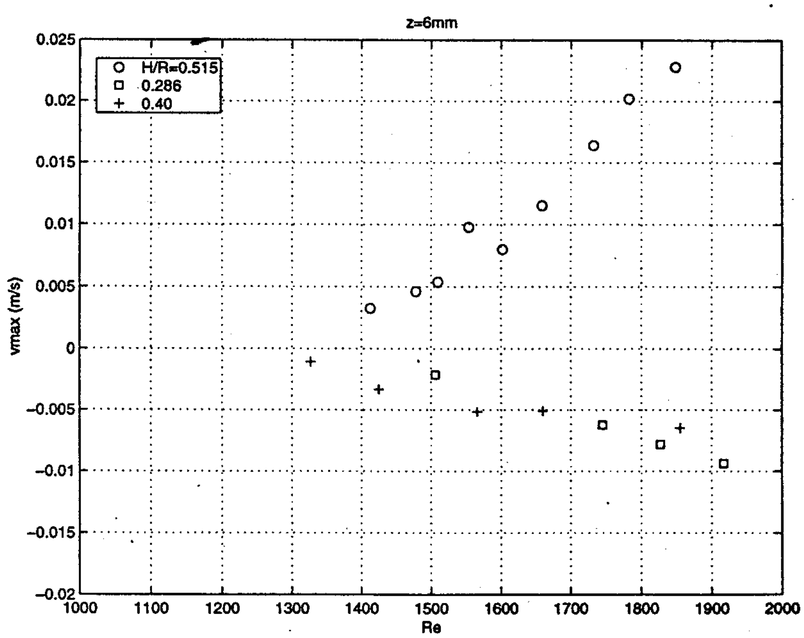

FIG. 4. Maximum swirl vs Re for several values of H/R and $z=6$ mm.

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smaller than the value obtained by Kawakubo *et al.*[5] in a open sink flow of water: From their Fig. 3, the critical Reynolds number for the appearance of swirl is about 1590. The present results suggest that the formation of a vortex in a confined sink flow constitutes a supercritical bifurcation, with approximately the same critical Reynolds number for the appearance and for the disappearance of the swirl. The present confined geometry has the advantage, in relation to an open sink flow, that the results can be much more easily checked by numerical simulation of the flow and, more importantly, the stability of the nonrotating sink flow can be much more easily analyzed. These are the subjects of future work.

ACKNOWLEDGMENT

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