

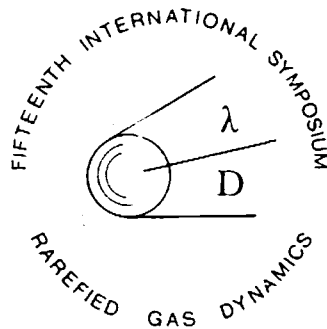
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TWO-FLUID THEORY FOR MONOATOMIC GASES AND
THE PROPAGATION OF SOUND IN BINARY MIXTURES
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Summary: A two-fluid Chapman-Enskog theory valid for arbitrary molecular interactions, and its application to the acoustic problem are presented. Comparison is made with experiments on sound absorption and dispersion in He-Xe mixtures.

1. Two-fluid Chapman-Enskog theory

In the classical Chapman-Enskog (C-E) procedure of solving the Boltzmann equations for a binary gas mixture, which schematically may be written as

$$D\phi_i = J_{ij}(f_j, f_i) + J_{ji}(f_i, f_j), \quad i=1,2, \quad j \neq i, \quad (1)$$

the velocity distribution function f_i of each species i is expanded around the local equilibrium distribution f_{0i} with a common temperature T and mean velocity U ,

$$f_{0i} = (m_i/2kT\pi)^{3/2} \exp[-m_i|u-U|^2/2kT]. \quad (2)$$

This method is appropriate when the time of equilibration between mean velocities and temperatures of each species by collisions, as well as the mean free path and time between molecular encounters of equal molecules, are very small compared to the characteristic macroscopic length and time of the system. If the species of the mixture have very different molecular masses m_i (say, $m_1 \ll m_2$), the inefficiency of collisions between unlike molecules in transferring momentum and energy with respect to collisions between identical molecules, results in that the former of the above conditions is not satisfied in most cases of practical interest. In terms of the species Knudsen numbers $Kn_i = \omega\mu_i/p_i$ (ω is a macroscopic characteristic frequency of the system, μ_i and p_i are the viscosity and partial pressure of species i), if both partial pressures p_i ($i=1,2$) are of the same order of magnitude, the usual C-E method is only valid when $Kn/M \ll 1$ ($M = m_1/m_2 \ll 1$) or, in other terms, when $\omega\tau \ll 1$, where τ is the slow relaxation time between species which is a quantity of order μ_i/Mp_i . For concreteness, τ will be defined by Einstein's law

$$\tau = D_{12}m_2(n_1+n_2)/kTn_1, \quad (3)$$

where D_{12} is the binary diffusion coefficient, n_i the number density of species i and T the mixture temperature.

To extend the theory to the same range of validity $Kn_i \ll 1$ [or $\omega\tau = O(1)$] of the C-E theory for binary gas mixtures with similar molecular masses, temperature and velocity differences between species have to be allowed. This, of course, is a well established fact and several two-fluid theories have been proposed in the past (i.e., Refs.[12],[10],[8]). Recently, we have developed [4] a two-fluid C-E theory, valid in first order in the temperatures and velocity differences, by rigorous expansion of the distribution function f_i of each species around its own local equilibrium distribution f'_{0i} with temperature T_i and mean velocity U_i :

$$f_i = f'_{0i} (1 + \phi_i' + \dots)$$

$$f'_{0i} = (m_i/2kT_i\pi)^{3/2} \exp[-m_i |u-U_i|^2/2kT_i]. \quad (4)$$

The substitution of these expressions into the Boltzmann equations yields linear integral equations for the first order corrections ϕ'_i . This was, in essence, the method of Goldman and Sirovich [10], who solved the problem for the case of Maxwellian molecules (fifth power law of intermolecular interaction).

Expansion (4) has the difficulty that f'_{0i} is not a solution of the right-hand sides (collision operators) of the Boltzmann Eq.(1), and extra terms of the form $J_{ij}(f'_{0i}, f'_{0j})$ appear. For this reason we carry out the calculation in terms of the functions f_i where

$$f_i = f_{0i} (1 + \phi_i + \dots) \quad (5)$$

and f_{0i} are the Maxwellian distributions (2) with a common mean velocity and temperature. The linear integral equations for f_i are the standard ones of the C-E theory for binary mixtures [3]

$$\begin{aligned} Df_i &= K_{11}(\phi_1) + K_{12}(\phi_1 + \phi_2) \\ Df_2 &= K_{21}(\phi_1 + \phi_2) + K_{22}(\phi_2) \end{aligned} \quad (6)$$

where the K_{ij} are the linearized Boltzmann collision operators; however, the determination of the terms Df_i is not based on the Maxwellians f_{0i} of the C-E expansion but on f'_{0i} ($Df_i = Df'_{0i}$). The resulting left-hand sides thus involve the spatial gradients of the species variables T_i, U_i , rather than the mixture variables T and U , as well as additional terms from the nonconservation of momentum and energy of each species separately. An additional substantial difference with the C-E theory is that it is now required that $f'_{0i}\phi'_i$ does not contribute to the temperature T_i and mean velocity U_i of the species i , instead of the C-E constraint that $f_{0i}\phi_i$ have null T and U . In terms of the standard inner product $[f_1(u), f_2(u)] = \int d^3u f_1(u) f_2(u)$, this means that $[1, f'_{0i}\phi'_i] = [u, f'_{0i}\phi'_i] = [u^2, f'_{0i}\phi'_i] = 0$, $i = 1, 2$, or, equivalently, since $f'_{0i}\phi'_i = f_{0i} - f'_{0i} + f_{0i}\phi_i + \dots$, in first order in the temperature and velocity differences, we have

$$\begin{aligned} [1, f_{0i}\phi_i] &= 0 \\ [u, f_{0i}\phi_i] &= n_i(U_i - U) \quad i = 1, 2 \\ [u^2, f_{0i}\phi_i] &= 3n_i k (T_i - T)/m_i. \end{aligned} \quad (7)$$

The details of the theory are given in Ref.[4], and we only outline here some of the results. Essentially, to satisfy the conditions (7), six free parameters in the vector function (ϕ_1, ϕ_2) are needed. As in the C-E theory, four of them are obtained by linear combinations of the collision invariants $(1, 0)$, $(0, 1)$, $(m_1 u, m_2 u)$ and $(m_1 u^2/2, m_2 u^2/2)$. These four parameters are enough to fix both number densities n_i ($i=1, 2$), the center of mass mean velocity U and the mixture temperature T . In the present theory we need two more parameters in order to allow for velocity and temperature differences. These two parameters are shown to be the momentum and energy transfer between species [4]

$$\begin{aligned} \lambda &= [m_1 u, K_{12}(\phi_1 + \phi_2)] \\ \sigma &= [m_1 u^2/2, K_{12}(\phi_1 + \phi_2)], \end{aligned} \quad (8)$$

which enter in the integral equations for ϕ_1 and ϕ_2 as a result of non-conservation of momentum and energy for each species separately. In fact, taking moments of the Boltzmann equation (1), the conservation equations for species i may be written as

$$\partial_t \rho_i + \nabla \cdot (\rho_i \mathbf{U}_i) = 0 \quad (9)$$

$$\partial_t (\rho_i \mathbf{U}_i) + \nabla \cdot (\rho_i \mathbf{U}_i \mathbf{U}_i + \mathbf{P}_i) = \lambda_i \quad (10)$$

$$\partial_t [\rho_i (e_i + U_i^2/2)] + \nabla \cdot [\rho_i \mathbf{U}_i (e_i + U_i^2/2) + \mathbf{P}_i \cdot \mathbf{U}_i + \mathbf{Q}_i] = \sigma_i \quad (11)$$

where $\rho_i \equiv [m_i, f_i]$, $\rho_i \mathbf{U}_i \equiv [m_i \mathbf{u}, f_i]$, $\mathbf{P} \equiv [m_i (\mathbf{u} - \mathbf{U}_i)(\mathbf{u} - \mathbf{U}_i), f_i]$, $\rho_i e_i \equiv \mathbf{P}_i : \mathbf{I}/2 \equiv 3n_i k T_i/2$, $\mathbf{Q}_i \equiv [m_i |\mathbf{u} - \mathbf{U}_i|^2 (\mathbf{u} - \mathbf{U}_i)/2, f_i]$, $\lambda_1 = -\lambda_2 = \lambda$ and $\sigma_1 = -\sigma_2 = \sigma$. Then, introducing expansion (5) into the Boltzmann equation and using (9-11), in first order in the temperature and velocity differences, the equations for ϕ_1 and ϕ_2 become [4]

$$\begin{aligned} K_{11} \phi_1 + K_{12} (\phi_1 + \phi_2) &= \text{Grad}_1 + f_{0i} [c \cdot \lambda + \sigma (m_1 c^2/3kT - 1)]/n_1 kT, \\ K_{21} (\phi_1 + \phi_2) + K_{22} \phi_2 &= \text{Grad}_2 \cdot f_{02} [c \cdot \lambda + \sigma (m_2 c^2/3kT - 1)]/n_2 kT, \end{aligned} \quad (12)$$

where Grad_i is the term appearing in the C-E theory for a pure gas

$$\text{Grad}_i = f_{0i} [m_i c \cdot \nabla \circ \mathbf{U}_i/kT + c \cdot \nabla \ln T_i (m_i c^2/2kT - 5/2)] \quad (13)$$

and $\mathbf{c} = \mathbf{u} - \mathbf{U}$. The other terms in the right-hand sides of Eqs.(12) result from the transfer of momentum and energy between species and contain the free parameters λ and σ . From the linear integral equations (12), using the constraints (7), it results[4]

$$\mathbf{P}_i = n_i k T_i - 2\mu_{i1} \nabla \circ \mathbf{U}_1 - 2\mu_{i2} \nabla \circ \mathbf{U}_2 \quad (14)$$

$$\mathbf{Q}_i = -\lambda_{i1} \nabla T_1 - \lambda_{i2} \nabla T_2 + n k T k_{Ti} (\mathbf{U}_1 - \mathbf{U}_2) \quad (15)$$

$$\lambda = -n k T (k_{T1} \nabla \ln T_1 + k_{T2} \nabla \ln T_2) - \rho_1 \rho_2 \lambda_b (\mathbf{U}_1 - \mathbf{U}_2)/m_2 \quad (16)$$

$$\sigma = -n^2 \sigma_T (T_1 - T_2). \quad (17)$$

The general expressions for the transport coefficients μ_{ij} , λ_{ij} , k_{Ti} , λ_b and σ_T , and the variational principles which can be used in their evaluation are given in Ref.[4]. The Appendix of the present work reports the calculated values in the first order of the Sonine polynomial expansion. All these coefficients, except σ_T , can be expressed in terms of the 'bracket' integrals appearing in the C-E theory [3]. In fact, $\mu = \mu_{11} + \mu_{22} + 2\mu_{12}$, $\lambda = \lambda_{11} + \lambda_{22} + 2\lambda_{12}$, $k_T = k_{T1} + k_{T2}$, and $\lambda_b = kT/m_1(n_1 + n_2)D_{12}$, where μ and λ are the mixture viscosity and thermal conductivity and k_T is the thermal diffusion ratio in the classical C-E theory. Thus the conservation equations (9-11) with (14-17) are equivalent to those of the classical C-E theory in the limit $T_1 = T_2 = T$, if we also make $\mathbf{U}_1 = \mathbf{U}_2 = \mathbf{U}$ in Eq.(14). The coefficient σ_T obviously does not enter the C-E theory but, to first order in the Sonine polynomial expansion, it can be related to D_{12} by $[\sigma_T]_1 = 3k^2 T x_1 x_2 / n(m_1 + m_2)[D_{12}]_1$, where $n \equiv n_1 + n_2$ and $x_i \equiv n_i/n$.

It must be noticed that the above results are valid for arbitrary intermolecular interactions. Moreover, there is no restriction with respect to the mass ratio M .

2. Application to sound propagation in binary mixtures

It has become a tradition to first test any two-fluid theory on the acoustic problem. The reasons are, obviously, its simplicity and the availability of experimental data. Thus, the two-fluid C-E theory of Goldman and Sirovich [10] was applied to the acoustic problem by Goldman [11] and compared with He-Ar absorption experimental data of Prangma et al.[17]. Although Goldman and Sirovich developed their two-fluid theory for Maxwellian molecules, Goldman in Ref. [11] uses

realistic values for the resulting transport coefficients and introduces 'ad hoc' the thermal diffusion which is missing in this model, so that his predictions are practically equivalent to ours. The two-fluid theory of Goebel et al.[9] (based on Grad's thirteen moments approximation) has been used by Huck and Johnson [14,15] and subsequently compared by Bowler and Johnson [1,2] with He-Xe experimental data. They predicted two modes of sound propagation for moderately high frequencies but such that $\omega\tau = O(1)$ and, thus, within the range of validity of the theory. Similar features were predicted recently by Fernandez de la Mora and Puri [5] by means of a simpler Euler-level two-fluid theory (that is, putting $P_i = n_i k T_i$ and $Q_i = 0$ in Eqs(9)-(11)), obtaining excellent agreement with He-Xe experimental dispersion data for $x_1 > 0.6$.

The present two-fluid theory, which we are about to test with He-Xe dispersion and absorption experimental data, is a natural extension of the above two-fluid Euler-level calculation to the next order approximation (Navier-Stokes-level) within the framework of the C-E procedure, to include viscous, heat flux and thermal diffusion effects. As pointed out in Refs.[16] and [5], except for the heat flux in the light gas energy equation, all these terms are, at most, of order $M\omega\tau$ (- Knudsen number) with respect to the leading terms in the corresponding Euler-level equations and, consequently, they only introduce small corrections of order M when $\omega\tau = O(1)$. However, the heat flux due to the light gas temperature gradients in the light gas energy equation, is of order $(Mx_1 + x_2)\omega\tau$ with respect to the first term in Eq.(11) and introduces corrections of order $\omega\tau$ when the heavy gas molar fraction x_2 is not small. Then, the inclusion of the dissipative effects in the Euler-level equations must correct significantly its predictions on the absorption of sound up to frequencies $\omega\tau = O(1)$. However, similarly to the classical Kohler analysis of sound propagation in binary mixtures (see, e.g., Ref.[7]), since dispersion is a quadratic effect in the frequency, to obtain a rigorous higher order correction to the dispersion given by the Euler-level theory, one must also take into account the next order (Burnett-level) of the C-E procedure in order to include terms of order $(\omega\tau M)^2$ (an expansion of the original Boltzmann kinetic equations in the frequency, similar to that of Ref.[7], is limited to $\omega\tau \ll 1$). Nevertheless, the dispersion predicted by the Euler-level theory (Ref.[5] and Fig.2 of the present work) is quite satisfactory (up to frequencies $\omega\tau = O(1)$) when $x_1 < 0.6$. Moreover, as we shall see, in the region of weaker dispersion ($x_1 > 0.6$) where the Euler-level dispersion predictions fail, the dispersion of sound obtained with the present two-fluid theory agrees rather well with experimental data.

In the acoustic limit, we linearize the conservation equations (9-11) and (14-17) around the unperturbed densities ρ_{10}, ρ_{20} , velocities $U_{10}=U_{20}=0$ and temperature T_0 . The perturbed hydrodynamic quantities are assumed to be plane waves propagating in the x direction, that is

$$\Psi = \Psi_0 + \Psi' \exp [i(\omega t - Kx)],$$

where Ψ represents any of the variables ρ_i, U_i (x -component of the velocity) and T_i ($i = 1,2$); ω is the frequency and K is the wave number. The resulting linearized conservation equations for the perturbed quantities Ψ' can be written, in a dimensionless compact form, as

$$(A + \zeta B) \cdot X = 0, \quad (18)$$

where X is the column vector formed with the dimensionless perturbed velocities and temperatures, $X^+ = (y_1, y_2, z_1, z_2)$, $y_1 \equiv U_1'/c$, $y_2 \equiv U_2'/c$, $z_1 \equiv T_1'/T_0$, $z_2 \equiv T_2'/T_0$, and $c \equiv \omega/K$ is the complex speed of sound; the rows of the matrices A and B are

$$\begin{aligned} & (M+x_2/\beta, -x_2/\beta, 0, 0) \\ & (-x_1/\beta, 1+x_1/\beta, 0, 0) \\ & (-(2/3)(k_{T1}/x_1+1), 2k_{T1}/3x_1, 1+E/\beta, -E/\beta) \end{aligned}$$

and

$$\begin{aligned}
 & (-2k_{T2}/3x_2, (2/3)(k_{T2}/x_2-1), -Ex_1/\beta x_2, 1+Ex_1/\beta x_2) \\
 & (-1-4M\beta S_{11}/3x_1, -4M\beta S_{12}/3x_1, -1-k_{T1}/x_1, -k_{T2}/x_1) \\
 & (-4M\beta S_{12}/3x_2, -1-4M\beta S_{22}/3x_2, k_{T1}/x_2, -1+k_{T2}/x_2) \\
 & (0, 0, -2\beta L_{11}/3x_1, -2\beta L_{12}/3x_1) \\
 & (0, 0, -2\beta L_{12}/3x_2, -2\beta L_{22}/3x_2)
 \end{aligned}$$

where $S_{ij} = \lambda_b \mu_{ij} / kT_0$, $L_{ij} = \lambda_b \lambda_{ij} m_i / k^2 T_0$, $E = 2\sigma_T / 3x_1 M k \lambda_b$. All the transport coefficients are calculated at the unperturbed temperature T_0 and molar fractions x_i . β is the dimensionless frequency

$$\beta = i\omega x_1 m_2 / \lambda_b \rho_{10} \quad (21)$$

and ζ , the eigenvalue in Eq.(23), is

$$\zeta = kT_0 / c^2 m_2. \quad (22)$$

The condition for the existence of nontrivial solutions of Eq.(18). i.e., $\det[A + \zeta B] = 0$, yields the complex eigenvalues ζ as functions of β and, thus, the dispersion relation $c=c(\omega)$. For real frequencies ω , the imaginary part of c is related to the absorption coefficient α , $\alpha = \text{Im}(-K) = \text{Im}(-\omega/c)$, while the real part is related to the speed of sound propagation c_R by $c_R^{-1} = \text{Re}(c^{-1})$.

2.1 Low frequency limit

Using Eq.(3) for τ with D_{12} given by the first order approximation in the Sonine polynomial expansion (Ref.[3] Eq. (9.81,1)), that is

$$\tau = 3/16 M n_{10} \Omega_{10}^{(1,1)}, \quad (23)$$

the parameter β may be rewritten as

$$\beta = i\omega \tau x_1 / a_0 = i s / a_0 \quad (24)$$

with $a_0 = 3\lambda_b / 16\Omega_{12}^{(1,1)} = 1$. In the limit $s = \omega \tau x_1 \ll 1$, we can solve Eq.(18) by expansion in the small parameter β :

$$\begin{aligned}
 X &= X_0 + \beta X_1 + \beta^2 X_2 + \dots \\
 \zeta &= \zeta_0 + \beta \zeta_1 + \beta^2 \zeta_2 + \dots
 \end{aligned} \quad (25)$$

At the zeroth order we obtain

$$\zeta_0 = 3(Mx_1 + x_2)/5 \quad (26)$$

which, obviously, corresponds to the equilibrium sound speed of the mixture (Laplace expression for low frequencies):

$$c_e^2 = 5kT_0 M / 3m_1 (Mx_1 + x_2) = 5p_0 / 3\rho_0. \quad (27)$$

In the first and second order, we have

$$-\zeta_1/\zeta_0 = 4MS/5 + 4(Mx_1 + x_2)L/25 + x_1x_2(M-1-2\alpha_T\zeta_0/3)^2/(Mx_1+x_2) \quad (28)$$

$$-5\zeta_2/3\zeta_0 = \zeta_1\Delta y_1x_2/\zeta_0 + 4M[(S_{22}+S_{12})\Delta y_1 + S\zeta_1/\zeta_0]/3 \\ + x_1x_2\Delta y_1\Delta y_2/\zeta_0 + \zeta_1Z_1/\zeta_0 + 2L(\zeta_0Z_1 + 2\zeta_1/3\zeta_0)/3. \quad (29)$$

where $S = S_{11} + S_{22} + 2S_{12}$, $L = L_{11} + L_{22} + 2L_{12}$, $\alpha_T = (k_{T1} + k_{T2})/x_1x_2$, $\Delta y_1 = M-1-2\zeta_0\alpha_T/3$, $Z_1 = 4(Mx_1 + x_2)L/15 + 2\Delta y_1x_2(1-x_1\alpha_T)/3$, and $-x_2\Delta y_2 = 5z_1/3 + 4M\zeta_0(S_{11}+S_{12})/3x_1 + (1 + x_2\alpha_T)\zeta_0Z_1 + 2\zeta_1\alpha_Tx_2/3$. If we write the real and imaginary part of the expansion (25) as

$$c_e/c_R \equiv \text{Re}(\zeta/\zeta_0)^{1/2} = 1 - d_2s^2 + \dots \\ \alpha c_e/\omega \equiv -\text{Im}(\zeta/\zeta_0)^{1/2} = d_1s + \dots$$

the coefficients d_1 and d_2 can be related to ζ_1 and ζ_2 by

$$d_1 = -\zeta_1/2a_0\zeta_0 \quad (30)$$

$$d_2 = [\zeta_2/\zeta_0 - (\zeta_1/2\zeta_0)^2]/2a_0^2. \quad (31)$$

Equations (30) and (28) yield Kohler's expression (see, e.g., Ref.[7]) for the absorption coefficient at low frequencies, derived from the classical C-E theory. This is not surprising because, in this limit $\omega\tau \ll 1$, the classical C-E theory is correct to first order in the frequency.

The other three eigenvalues of Eq.(18) are non-propagating at low frequencies since they tend to infinity as the frequency (β) goes to zero. In particular, two of them behave as $\zeta \sim \zeta'_0/\beta$ for $\beta \ll 1$, with ζ'_0 given by the two roots of $\zeta'_0(\zeta'_0-1)L - k_T^2\zeta'_0/x_1x_2 - 5(\zeta'_0-1)/2 = 0$. Neglecting the thermal diffusion term in this equation, one obtains $\zeta'_0 = 1$ and $\zeta'_0 = 5/2L$ which, in physical variables, yield the low frequency dispersion relations $K^2 = -i\omega(n_{10}+n_{20})(5k/2\lambda)$ and $K^2 = -i\omega/D_{12}$, corresponding to the thermal and diffusion modes, respectively (which in fact are weakly mixed through thermal diffusion). The third non-propagating eigenvalue behaves as $\zeta \sim \zeta_0''/\beta^2$ for $\beta \ll 1$, where $\zeta_0'' = 3ELx_1/2(L_1L_2 - L_{12}^2)$.

3. Comparison with experiments and discussion.

In Figs. 1 and 2 we compare the numerical solution of Eq.(18) which at low frequencies is given by Eqs.(25), with experimental data in He-Xe taken from the Ph.D.thesis of Bowler [1] and that of Fuentes Losa [8]. Notice that the labels of x_1 in the experimental data of Fuentes Losa are based on the extrapolated sound velocity at zero frequency using Eq.(27) (see Refs. [5] and [6]). All the numerical results are computed at $T_0=300\text{K}$, $M = 4/131.3$ and using a Lennard-Jones intermolecular potential (see, i.e., Ref.[13]; we have taken from this reference the values of σ_i and ϵ_i while the values of σ_{12} and ϵ_{12} are from Ref.[14]). For some values of the light gas molar fraction x_1 , the Euler-level solution is included.

Fig.1 contains the comparison with the absorption coefficients ($\alpha c_e/\omega$) obtained experimentally by Bowler. We note that the agreement is very good up to frequencies $\omega/p=60 \text{ MHz/Atm}$ (or even

more for some values of x_1), which corresponds to $s \equiv \omega \tau x_1 \approx 1$ and, thus, is within our expectations. In Fig.2 we plot c_1/c_R (where c_1 is the light gas equilibrium sound speed, $c_1^2 \equiv 5kT_0/3m_1$) versus the squared dimensionless frequency, s^2 . Notice that the slope of the curves at $s^2=0$ is $c_1 d_2/c_e$, where d_2 is given by Eq.(31). Remarkably enough, despite the fact that a rigorous correction of the Euler-level dispersion must include the next (Burnett) C-E approximation, for the values of x_1 where the Euler theory is poorer ($x_1 < 0.6$, specially $x_1 < 0.3$), agreement with experiments is quite good up to $s = 0(1)$. However, for low heavy gas concentrations, particularly for x_1 between 0.7 and 0.95, the Euler-level predictions seem to be better. For x_1 near unity ($x_1 > 0.95$), both theories almost coincide and agree with the experiments.

In Figs. 1 and 2 only that eigenvalue of Eq.(18) which is physically relevant at low frequencies (sound root) is represented (except for $x_1 = .45$ and $.5$ for which the diffusion mode is also included).

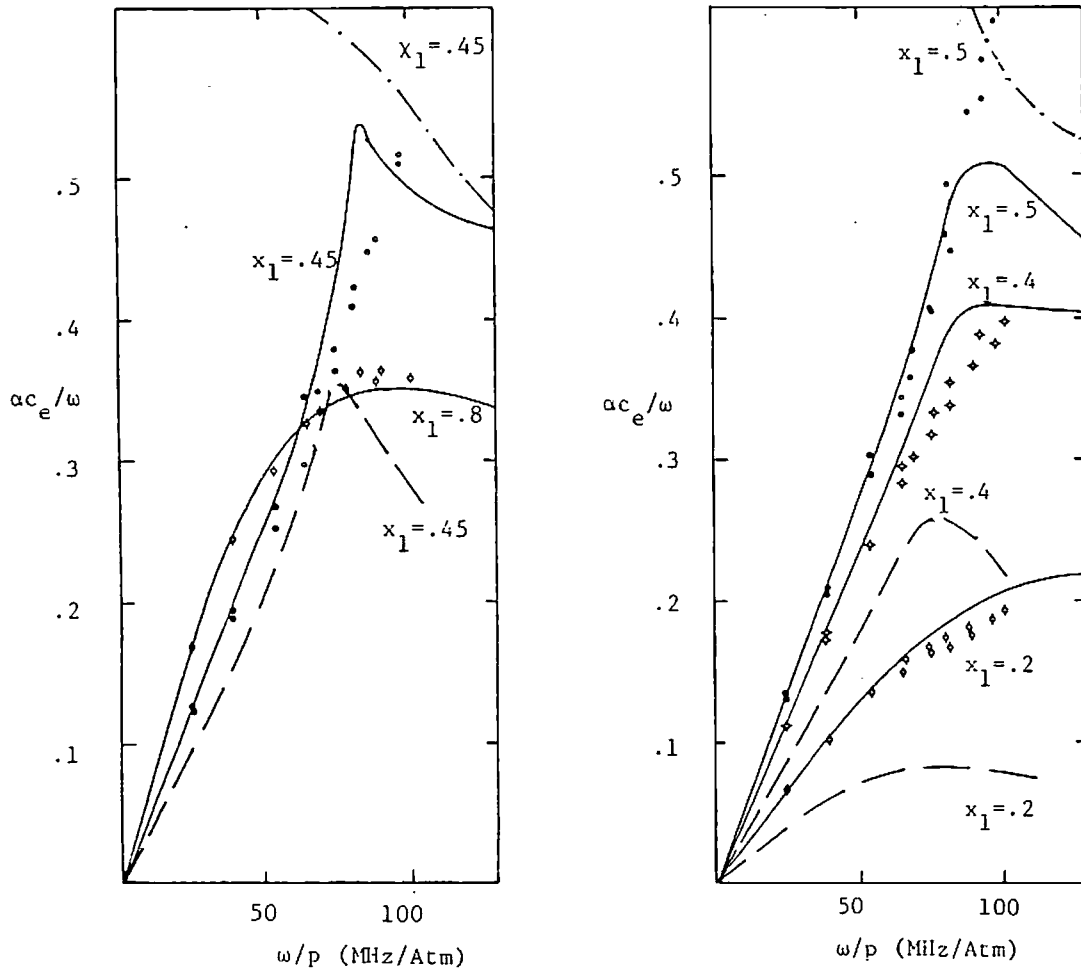


Fig.1 Absorption in He-Xe for some values of the He molar fraction. — :sound root; - - -:sound root in the Euler-level; - · - · :diffusion root; · ♦ · experimental data of Bowler [1].

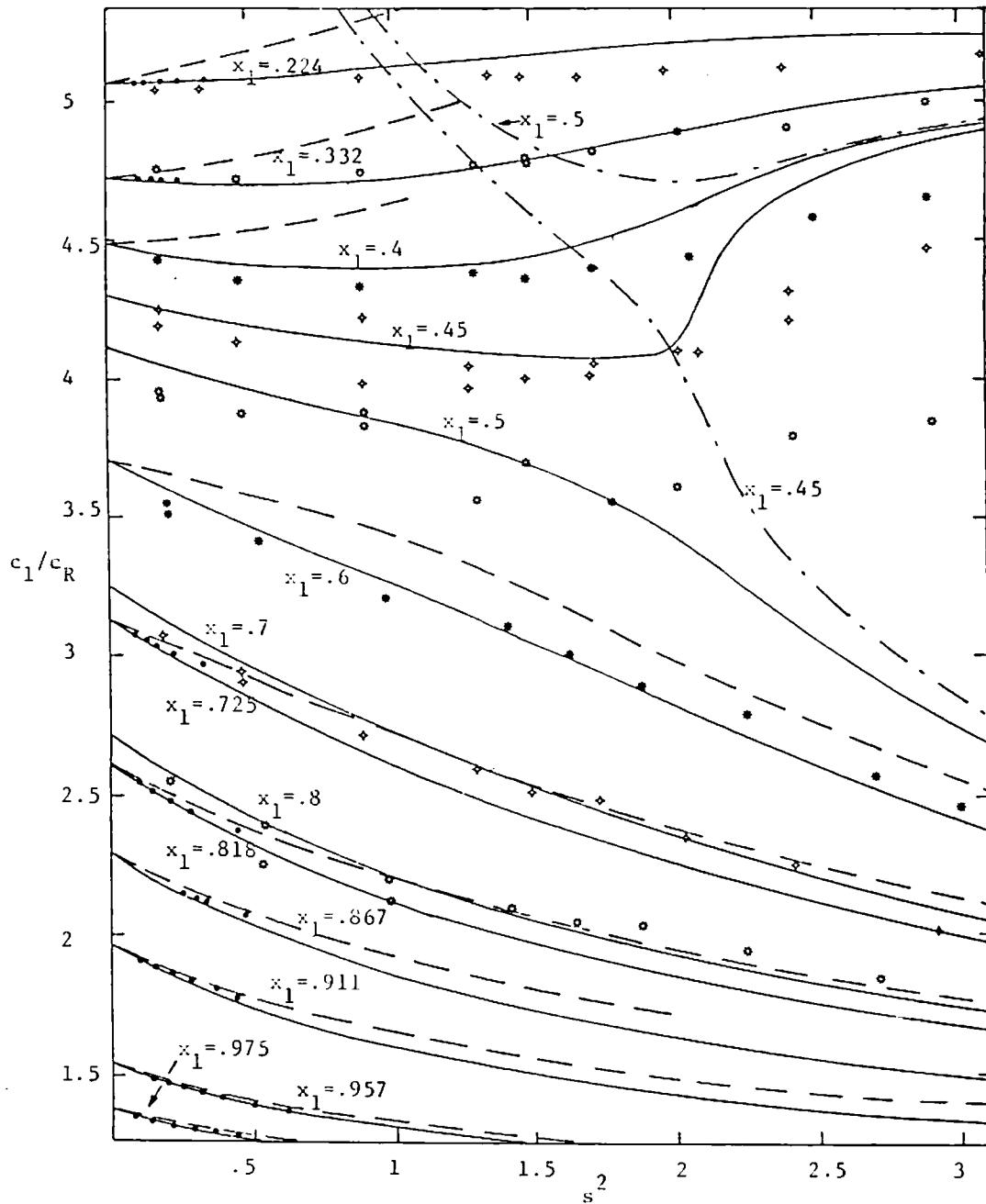


Fig.2 Dimensionless sound speed c_1/c_R versus dimensionless frequency s in He-Xe for some values of He molar fraction. — :sound root; - - :sound root in the Euler-level; - · - :diffusion root; • :experimental data of Fuentes Losa [8]; ♦ ♦ ♦ :experimental data of Bowler [1] (from top to bottom, $x_1 = .2, .3, .4, .45, .5, .6, .7, .8$).

As pointed out by Huck and Johnson [15,16], for frequencies larger than a critical value (for He-Xe at 300K we obtain $\omega_c \approx 2.2/\tau x_{1c}$, with the critical light gas molar fraction x_{1c} between .450 and .455) there is another eigenvalue of Eq.(18) which must be taken into account. Under the present conditions, this other eigenvalue becomes the diffusion mode at low frequencies (see the end of previous section). This seems reasonable since at the Euler-level of the theory, at which the dispersion is so well predicted for $x_1 > 0.6$, it is also the diffusion mode which interferes for $\omega > \omega_c$ [5] (the thermal mode is obviously missing at the Euler-level). Bowler and Johnson [11], using Grad's thirteen moment approximation with Maxwell molecules model, obtained the thermal mode as the interfering one. However, using other intermolecular potentials, but still Grad's method, they also obtained the diffusion mode (private communication).

In conclusion, the two-fluid theory here presented predicts satisfactorily the absorption of sound in He-Xe mixtures up to frequencies $\omega\tau = 0(1)$, improving the classical C-E theory (Kohler) valid for $\omega\tau \ll 1$. As expected, it corrects rather well the poor absorption predictions of the Euler-level. For dispersion, a higher order theory including two-fluid Burnett corrections is needed. However, the predictions at the present level compare quite well with experiments. In relation to previous works, the present two-fluid theory uses a more realistic potential of molecular interaction (Lennard-Jones molecular model vs. the Maxwell or hard spheres model; see, however, previous comment on Goldman's work). The present approach results from a rigorous asymptotic expansion rather than from Grad's thirteen moments. However, it is limited (as previous two-fluid theories) to small slip in the velocities and temperatures with respect to the characteristic (sound) velocity and temperature (which, nevertheless, is a fairly good assumption in the acoustic problem).

Appendix. Expressions for the transport coefficients in the first order of the Sonine polynomial expansion are listed below. The Ω -integrals for the Lennard-Jones potential are tabulated, i.e., in Ref.[13] pp.1126-27.

$$[\mu_{11}]_1 = [x_1^2 (2/3 + M_1 A_1/M_2) + x_1 x_2 B_1/2\mu_2]/Q_1$$

$$[\mu_{22}]_1 = [x_2^2 (2/3 + M_2 A_1/M_1) + x_1 x_2 B_1/2\mu_1]/Q_1$$

$$[\mu_{12}]_1 = [\mu_{21}]_1 = x_1 x_2 (2/3 - A_1)/Q_1$$

$$[\lambda_{11}]_1 = (75 x_1 k^2 T/64 m_1 Q_2) (P_2 - P_1 P_7 / P_6)$$

$$[\lambda_{22}]_1 = (75 x_2 k^2 T/64 m_2 Q_2) (P_5 - P_1 P_8 / P_6)$$

$$[\lambda_{12}]_1 = [\lambda_{21}]_1 = - (75 x_2 k^2 T/ 64 m_2 Q_2) (P_3 + P_1 P_8 / P_6)$$

$$[k_{T1}]_1 = - 5 M_2 x_1 x_2 P_7 / 2 x_0 P_6.$$

$$[k_{T2}]_1 = - 5 M_1 x_2^2 P_8 / 2 x_0 P_6$$

$$[\lambda_b]_1 = - 16 M_2 Q_2 / 3 x_0 P_6.$$

$$[\sigma_{\tau}]_1 = 16 k M_1 M_2 x_1 x_2 \Omega_{12}^{(1,2)},$$

where

$$Q_1 = x_1^2 (2/3 + M_1 A_1/M_2)/\mu_1 + x_2^2 (2/3 + M_2 A_1/M_1)/\mu_2 + x_1 x_2 (B_1/2 \mu_1 \mu_2 + 4A/3B_1 M_1 M_2)$$

$$\mu_1 = 5kT/8\Omega_1^{(2,2)},$$

$$\mu_2 = 5kT/8\Omega_2^{(2,2)}$$

$$A_1 = \Omega_{12}^{(2,2)}/5 \Omega_{12}^{(1,1)},$$

$$B_1 = kT/8M_1 M_2 \Omega_{12}^{(1,1)}$$

$$Q_2 = BM_2 x_0 P_1 + (x_1 E + M_2 x_2 C) P_2 + M_1^2 M_2 x_2 G P_3$$

$$P_1 = M_1^{3/2} M_2^{3/2} B (x_2 F/x_1 M_1 + D - M_1^2 G)$$

$$P_2 = M_1^{1/2} x_0 [M_1 (M_1^2 B^2 - A D)/x_2 - A F/x_1]$$

$$P_3 = (M_1 M_2)^{3/2} (A G - B^2) x_0 /x_2$$

$$P_5 = M_1^{1/2} x_0 [M_2 (M_2^2 B^2 - A C)/x_1 - A E/x_2]$$

$$P_6 = M_1^{1/2} [(x_1 E/x_2 + M_2 C) (x_2 F/x_1 + M_1 D) - M_1^3 M_2^3 G^2]$$

$$P_7 = M_1^{1/2} x_0 B [M_2 F/x_1 + M_1 M_2 (D - M_1^2 G)/x_2]$$

$$\begin{aligned}
 P_8 &= M^{1/2} x_0 B [E x_1 M_1/x_2 + M_1 M_2 (C - M_2^2 G)] / x_2 \\
 A &= \Omega_{12}^{(1,1)}, & B &= 5\Omega_{12}^{(1,1)}/2 - \Omega_{12}^{(1,2)} \\
 C &= 5(6M_1^2 + 5M_2^2)\Omega_{12}^{(1,1)}/4 - 5M_2^2\Omega_{12}^{(1,2)} + M_2^2\Omega_{12}^{(1,3)} + 2M_1M_2\Omega_{12}^{(2,2)} \\
 D &= 5(6M_2^2 + 5M_1^2)\Omega_{12}^{(1,1)}/4 - 5M_1^2\Omega_{12}^{(1,2)} + M_1^2\Omega_{12}^{(1,3)} + 2M_1M_2\Omega_{12}^{(2,2)} \\
 E &= \Omega_1^{(2,2)}/2, & F &= \Omega_2^{(2,2)}/2 \\
 G &= 55\Omega_{12}^{(1,1)}/4 - 5\Omega_{12}^{(1,2)} + \Omega_{12}^{(1,3)} - 2\Omega_{12}^{(2,2)} \\
 x_0 &= M_1x_1 + M_2x_2, & M &= m_1/m_2, & M_i &= m_i/(m_1+m_2).
 \end{aligned}$$

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References

- [1] Bowler, J.R.: Ph.D. Thesis, University of Surrey, U.K., (1984) (Unpublished).
- [2] Bowler, J.R.; Johnson, E.A.: Phys. Rev. Lett. 54 (1985) 329
- [3] Chapman, S.; Cowling, T.G.: "The Mathematical Theory of Non-Uniform Gases". Cambridge University Press (1970)
- [4] Fernandez de la Mora, J.; Fernandez-Feria, R.: "Two-Fluid Chapman-Eskog Theory". To be published.
- [5] Fernandez de la Mora, J.; Puri, A.: J. Fluid Mech. 68 (1986) 369
- [6] Fernandez de la Mora, J.: J. Phys. Chem. 88 (1984) 4557
- [7] Foch, J.D.; Uhlenbeck, G.E.; Fuentes Losa, M.: Phys. Fluids 15 (1972) 1224
- [8] Fuentes Losa, M.: Ph.D. Thesis, University of Colorado (1972)
- [9] Goebel, C.G.; Harris, S.M.; Johnson, E.A.: Phys. Fluids 19 (1976) 617
- [10] Goldman, E.B.; Sirovich, L.: Phys. Fluids 10 (1967) 1928
- [11] Goldman, E.B.: Journal of Acoustic Soc. Am. 44 (1968) 708
- [12] Grad, H.: in Rarefied Gas Dynamics, F.M. Devienne, ed. Pergamon: N.Y. (1960); Hamel, B.B.: Phys. Fluids 9 (1966), 12
- [13] Hirschfelder, J.O.; Curtiss, C.F.; Bird, R.B.: "Molecular Theory of Gases and Liquids". Wiley (1958)
- [14] Hogervorst, W.: Physica 51 (1971) 59-103
- [15] Huck, R.J.; Johnson, E.A.: Phys. Rev. Lett. 44 (1980)
- [16] Huck, R.J.; Johnson, E.A.: in Rarefied Gas Dynamics, S.S. Fisher, ed., American Institute of Aeronautics and Astronautics New York. (1980) 452-463
- [17] Prangma, G.J.; Jonkman, R.M.; Beenakker, J.: Physica 48 (1970) 323.

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